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Effects of Hall current and heat transfer on flow due to a pull of eccentric rotating disks

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Abstract

The influence of Hall current and heat transfer on the magnetohydrodynamic (MHD) flow of an Oldroyd-B fluid is investigated. The fluid is between two infinite disks rotating about non-coaxial axes normal to the disks in the presence of a uniform transverse magnetic field. The flow is due to a constant velocities of eccentric rotating disks. Exact solutions are derived for the velocity, force and torque exerted by the fluid on one of the disk and temperature distribution.

1. Introduction

In recent years the theoretical study of MHD channel flows has been a subject of great interest due to its widespread applications in designing cooling systems with liquid metals, petroleum industry, purification of crude oil, polymer technology, centrifugal separation of matter from fluid, MHD generators, pumps, accelerators and flow meters. Various workers [1–6] have analyzed the interesting problems in this direction. More recently, Ersoy [7] discussed the flow due to a pull with constant velocities of eccentric rotating disks with the same angular velocity. In another paper Ersoy [8] examined the MHD flow of a non-Newtonian fluid between eccentric rotating disks.

Unfortunately, the results of the above investigations cannot be applied to the flow of ionized gases. In an ionized gas where the density is low and/or the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions; also, a current is induced in a direction normal to both the electric and magnetic fields. The phenomena, well known in the literature, is called the Hall effect. The study of magnetohydrodynamic flows with Hall currents has important engineering applications in problems of magnetohydrodynamic generators and of Hall accelerators as well as in flight magnetohydrodynamics. Sato [9], Sherman and Sutton [10], Hossain [11], Hossain and Mohammad [12], Hossain and Rashid [13], Pop [14], Raptis and Ram [15] and Ram [16] studied the Hall effects.

In this paper, we generalize the work of Ersoy [7,8] and investigate the effects of Hall current and heat transfer on the steady flow of an electrically conducting, Oldroyd-B and incompressible fluid. The fluid is between two electrically insulating disks maintained at two constant but different temperatures. The flow is due to a pull with constant velocities of eccentric rotating infinite disks and an external uniform magnetic field is applied

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Nomenclature

$\mathbf{A}_1$	Rivlin-Ericksen tensors
B_0	applied magnetic field
C_p	specific heat at constant pressure
$\mathrm{D}/\mathrm{D}t$	upper convected derivative
e	electric charge
$\mathbf{E}$	total electric field current
E_c	Eckert number
$\mathbf{I}$	unit tensor
M	Hartmann number
n_e	number of density of electrons
p	pressure
p_e	electron pressure
p_0	reference pressure
$\mathbf{q}$	heat flux vector
r_0	radius of the disk
R	Reynolds number
$\mathbf{r}$	radiant heating
$\mathbf{S}$	deviatoric stress tensor
T	transpose
$\mathbf{T}$	Cauchy stress tensor
u, v, w	velocity components
U_1	disk velocity in x-direction
U_2	disk velocity in y-direction
$\mathbf{V}$	velocity vector
x, y, z	coordinate axis
$\mathbf{X}, \mathbf{Y}, \mathbf{Z}$	components of the force

Greek symbols

α	elastic number
α_1	elasticity
α_2	cross-viscosity
θ	dimensionless temperature
λ	surface of the disk
λ_1	relaxation time
λ_2	retardation time
μ	dynamic viscosity
μ_m	magnetic permeability
v	kinematic viscosity
ρ	density
σ	electrical conductivity
τ_e	electron collision time
τ_i	ions collision time
τ_1	temperature at the lower disk
τ_2	temperature at the upper disk
τ	temperature
ϕ	Hall parameter
χ	torque
ψ	stream function
ω_i	cyclotron frequency of ions
ω_e	cyclotron frequency of electrons
ω	vorticity vector
Ω	angular velocity

perpendicular to the disks. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected. Firstly, the formulation and solution of the problem in regard to the velocity distribution is given. Secondly, the heat transfer characteristics are discussed and then the expressions for force and torque exerted by the fluid on the bottom disk are constructed. Finally the numerical discussions and conclusions are given.

2. Mathematical formulation

Let us consider two infinite disks at $z = h$ and $z = -h$ rotating around z-axes respectively with the same angular velocity Ω. The two non-coincident axes are separated by an instance $2l$. The region between the two disks is occupied by an incompressible and electrically conducting Oldroyd-B fluid. The magnetic field is applied perpendicular to the disks. The disks at $z = h$ and $z = -h$ are pulled with velocities $\mathbf{U}$ and $-\mathbf{U}$ respectively. Thus, the boundary conditions become

$$u = -\Omega(y - l) + U_1, \quad v = \Omega x + U_2, \quad w = 0 \quad \text{at } z = h, \tag{2.1}$$

$$u = -\Omega(y + l) - U_1, \quad v = \Omega x - U_2, \quad w = 0 \quad \text{at } z = -h. \tag{2.2}$$

The velocity field is chosen as

$$u = -\Omega y + f(z), \quad v = \Omega x + g(z), \quad w = 0. \tag{2.3}$$

The MHD equations governing the steady flow of an incompressible fluid are

$$\rho[(\mathbf{V}\cdot\nabla)\mathbf{V}] = \operatorname{div}\mathbf{T} + \mathbf{J}\times\mathbf{B}, \tag{2.4}$$

$$\nabla\cdot\mathbf{V} = 0, \tag{2.5}$$

$$\nabla\cdot\mathbf{B} = 0, \quad \nabla\times\mathbf{B} = \mu_m\mathbf{J}, \quad \nabla\times\mathbf{E} = 0 \tag{2.6}$$

in which ρ is the density, $\mathbf{V}$ is the velocity, $\mathbf{J}$ is the current density, $\mathbf{B}$ is the total magnetic field, μ_m the magnetic permeability, $\mathbf{E}$ the total electric field current and σ the electrical conductivity of the fluid. Making reference to Cowling [17], when the strength of the magnetic field is very large, the generalized Ohm's law is modified to include the Hall current so that

$$\mathbf{J} + \frac{\omega_e\tau_e}{\mathbf{B}_0}(\mathbf{J}\times\mathbf{B}) = \sigma\left[\mathbf{E} + \mathbf{V}\times\mathbf{B} + \frac{\mathbf{1}}{en_e}\nabla p_e\right], \tag{2.7}$$

where ω_e is the cyclotron frequency of electrons, τ_e is the electron collision time, e is the electron charge, n_e is the number of density of electrons, and p_e is the electron pressure. The ion-slip and thermoelectric effects are not included in (2.7). Further, it is assumed that $\omega_e\tau_e \approx O(1)$ and $\omega_i\tau_i \ll 1$ where ω_i and τ_i are the cyclotron frequency and collision time for ions respectively.

The constitutive equation of an Oldroyd-B fluid is

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S} \tag{2.8}$$

in which

$$\mathbf{S} + \lambda_1 \frac{\mathrm{D}\mathbf{S}}{\mathrm{D}t} = \mu\left[1 + \lambda_2 \frac{\mathrm{D}}{\mathrm{D}t}\right]\mathbf{A}_1 \tag{2.9}$$

with p being the pressure, $\mathbf{I}$ the unit tensor, $\mathbf{L}$ the gradient of the velocity vector, $\mathbf{S}$ the deviatoric stress tensor, μ the dynamic viscosity, λ_1 the relaxation time, λ_2 the retardation time, $\mathbf{A}_1$ is the Rivlin-Ericksen tensor defined by

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^{\mathrm{T}}, \quad \mathbf{L} = \nabla\mathbf{V}, \tag{2.10}$$

and T is the transpose. The upper convected time derivative D/Dt, operating on $\mathbf{S}$, is defined as

$$\frac{\mathrm{D}\mathbf{S}}{\mathrm{D}t} = \frac{\partial\mathbf{S}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{S} - \mathbf{S}\mathbf{L} - \mathbf{L}^{\mathrm{T}}\mathbf{S}. \tag{2.11}$$

When $\lambda_1 = \lambda_2 = 0$ Eq. (2.9) reduces to the classical linearly viscous model and when $\lambda_2 = 0$ Eq. (2.9) becomes a Maxwell model. For $\lambda_1 = 0$, $\mu\lambda_2 = \alpha_1$, it reduces to second-grade fluid.

As it is seen that the velocity field satisfies Eq. (2.5) which is nothing else than the incompressibility condition and from Eqs. (2.3), (2.9), (2.10) and (2.11) we have

$$s_{xx} = \mu\frac{2(\lambda_1-\lambda_2)\lfloor(1+\lambda_1^2\Omega^2)f'^2 + 3\lambda_1^2\Omega^2 g'^2 - 3\lambda_1 f'g'\rfloor}{(1+\lambda_1^2\Omega^2)(1+4\lambda_1^2\Omega^2)}, \tag{2.12}$$

$$s_{xy} = \mu\frac{(\lambda_1-\lambda_2)\lfloor 2(1-2\lambda_1^2\Omega^2)f'g' + 3\lambda_1\Omega(f'^2 - g'^2)\rfloor}{(1+\lambda_1^2\Omega^2)(1+4\lambda_1^2\Omega^2)}, \tag{2.13}$$

$$s_{yy} = \mu\frac{2(\lambda_1-\lambda_2)\lfloor 3\lambda_1\Omega f'(g' + \lambda_1\Omega f') + (1+\lambda_1^2\Omega^2)g'^2\rfloor}{(1+\lambda_1^2\Omega^2)(1+4\lambda_1^2\Omega^2)}, \tag{2.14}$$

$$s_{xz} = \mu\frac{(1+\lambda_1\lambda_2\Omega^2)f' + (\lambda_2-\lambda_1)\Omega g'}{(1+\lambda_1^2\Omega^2)}, \tag{2.15}$$

$$s_{yz} = \mu\frac{(1+\lambda_1\lambda_2\Omega^2)g' + (\lambda_1-\lambda_2)\Omega f'}{(1+\lambda_1^2\Omega^2)}, \tag{2.16}$$

$$s_{zz} = 0, \tag{2.17}$$

where we have assumed that the stress s also depends on z and satisfy $s_{ij} = s_{ij}(z)$. With the help of Eqs. (2.4) and (2.6)–(2.8) we can write the following scalar equations:

$$\frac{\partial p}{\partial x} = \rho\Omega[\Omega x + g(z)] + \frac{\partial}{\partial z}s_{xz} + \frac{\sigma B_0^2(1+\mathrm{i}\phi)}{1+\phi^2}\left(\frac{\overline{Q}}{2h} - f\right), \tag{2.18}$$

$$\frac{\partial p}{\partial y} = -\rho\Omega[-\Omega y + f(z)] + \frac{\partial}{\partial z}s_{yz} + \frac{\sigma B_0^2(1+\mathrm{i}\phi)}{1+\phi^2}\left(\frac{\overline{P}}{2h} - g\right), \tag{2.19}$$

$$\frac{\partial p}{\partial z} = 0, \tag{2.20}$$

$$\overline{P} = \int_{-h}^{h} g(z)\,\mathrm{d}z, \quad \overline{Q} = \int_{-h}^{h} f(z)\,\mathrm{d}z, \tag{2.21}$$

where $\phi = \omega_e\tau_e$ is the Hall parameter and Eq. (2.20) indicates that p is not a function of z.

The boundary conditions in terms of f and g can be written as

$$f(h) = \Omega l + U_1, \quad g(h) = U_2, \tag{2.22}$$

$$f(-h) = -\Omega l - U_1, \quad g(-h) = -U_2. \tag{2.23}$$

3. Exact solution

Differentiating Eqs. (2.12) and (2.13) with respect to z, making use of Eq. (2.14), and then integrating the resulting expression we obtain the following equations:

$$\frac{\partial}{\partial z}s_{xz} + \rho\Omega g - Hf = C_1, \tag{3.1}$$

$$\frac{\partial}{\partial z}s_{yz} - \rho\Omega f - Hg = C_2, \tag{3.2}$$

where C_1 and C_2 are arbitrary constants and

$$H = \frac{\sigma B_0^2(1+\mathrm{i}\phi)}{1+\phi^2}. \tag{3.3}$$

The pressure field is obtained by integrating Eqs. (2.12) and (2.13)

$$p = p_0 - \frac{1}{2}\Omega^2(x^2+y^2) + [C_1 + H\overline{Q}/2h]x + [C_2 + H\overline{P}/2h]y, \tag{3.4}$$

where p_0 is a reference pressure. We see from the above equation that non-zero values of $C_1 + H\overline{Q}/2h$ and $C_2 + H\overline{P}/2h$ would give rise to a pressure gradient between the disks with a corresponding Poiseuille type flow. In order to remove the possibility of a flow of this type and at the same time to ensure the symmetry of the velocity distribution about the disk $z = 0$, we put

$$C_1 = -H\overline{Q}/2h, \quad C_2 = -H\overline{P}/2h \tag{3.5}$$

and Eqs. (2.15), (2.16), (3.1) and (3.2) yield

$$\mu\frac{(1+\lambda_1\lambda_2\Omega^2)+\mathrm{i}(\lambda_1-\lambda_2)\Omega}{(1+\lambda_1^2\Omega^2)}F''-\mathrm{i}\Omega F-HF + \left(\frac{\overline{P}}{2h}+\frac{\overline{Q}}{2h}\right)H=0, \tag{3.6}$$

where $F=f+\mathrm{i}g$.

The boundary value problem in dimensionless variable takes the following form:

$$\Gamma''(\eta)-\frac{[M^2(1+\mathrm{i}\phi)+\mathrm{i}R(1+\phi^2)](1+D^2)}{[(1+\alpha D^2)+\mathrm{i}D(1-\alpha)](1+\phi^2)}\Gamma(\eta) = \frac{-M^2(1+\mathrm{i}\phi)(1+D^2)}{[(1+\alpha D^2)+\mathrm{i}D(1-\alpha)](1+\phi^2)}\frac{(\overline{Q}+\mathrm{i}\overline{P})}{2h\Omega l}, \tag{3.7}$$

$$\Gamma(1)=[(1+V_1)+\mathrm{i}V_2], \quad \Gamma(-1)=-[(1+V_1)+\mathrm{i}V_2], \tag{3.8}$$

where

$$\eta=z/h, \quad \Gamma(\eta)=F(z)/\Omega l=(f+\mathrm{i}g)/\Omega l,$$
$$\phi=\omega_e\tau_e, \quad R=\rho\Omega h^2/\mu,$$
$$M^2=\sigma B_0^2h^2/\mu, \quad D=\Omega\lambda_1, \quad \alpha=\lambda_2/\lambda_1,$$
$$V_1=\Omega lU_1, \quad V_2=\Omega lU_2,$$

where R is the Reynolds number, M is the Hartmann number, D is the Deborah number (measure of the fluid elasticity) and α the elastic number.

The solution of Eq. (3.7) satisfying the boundary conditions (3.8) is given by

$$\Gamma(\eta)=[(1+V_1)+\mathrm{i}V_2]\frac{\sinh m_1\eta}{\sinh m_1}, \tag{3.9}$$

where

$$m_1=\frac{[M^2(1+\mathrm{i}\phi)+\mathrm{i}R(1+\phi^2)](1+D^2)}{[(1+\alpha D^2)+\mathrm{i}D(1-\alpha)](1+\phi^2)}. \tag{3.10}$$

Equating real and imaginary parts of above expression we obtain

$$\frac{f}{\Omega l}=\frac{\left[\begin{array}{l}(1+V_1)\left(\begin{array}{l}\sinh\xi_1\eta\cos\xi_2\eta\sinh\xi_1\cos\xi_2\\ +\cosh\xi_1\eta\sin\xi_2\eta\cosh\xi_1\sin\xi_2\end{array}\right)\\ -V_2\left(\begin{array}{l}\cosh\xi_1\eta\sin\xi_2\eta\sinh\xi_1\cos\xi_2\\ -\sinh\xi_1\eta\cos\xi_2\eta\cosh\xi_1\sin\xi_2\end{array}\right)\end{array}\right]}{\Delta}, \tag{3.11}$$

$$\frac{g}{\Omega l}=\frac{\left[\begin{array}{l}(1+V_1)\left(\begin{array}{l}\cosh\xi_1\eta\sin\xi_2\eta\sinh\xi_1\cos\xi_2\\ -\sinh\xi_1\eta\cos\xi_2\eta\cosh\xi_1\sin\xi_2\end{array}\right)\\ -V_2\left(\begin{array}{l}\sinh\xi_1\eta\cos\xi_2\eta\sinh\xi_1\cos\xi_2\\ +\cosh\xi_1\eta\sin\xi_2\eta\cosh\xi_1\sin\xi_2\end{array}\right)\end{array}\right]}{\Delta}. \tag{3.12}$$

In above equations

$$\begin{aligned}
&\Delta=\sinh^2\xi_1\cos^2\xi_2+\cosh^2\xi_1\sin^2\xi_2,\\
&\xi_1=\sqrt{\frac{\sqrt{d_1^2+e_1^2}+d_1}{2}\,\frac{c}{a^2+b^2}},\\
&\xi_2=\sqrt{\frac{\sqrt{d_1^2+e_1^2}-d_1}{2}\,\frac{c}{a^2+b^2}},\\
&a=1+\alpha D^2, \quad b=D(1-\alpha),\\
&c=1+D^2, \quad \bar{a}=1+\phi^2,\\
&d_1=\frac{1}{\bar{a}}[M^2a+b(\phi M^2+R\bar{a})],\\
&e_1=\frac{1}{\bar{a}}[a(\phi M^2+R\bar{a})-M^2b].
\end{aligned} \tag{3.13}$$

4. Heat transfer

In this section we apply heat transfer analysis by assuming that the disks at $z=h$ and $z=-h$ are heated and that the heat is transferring from disks to the fluid. The law of conservation of energy is

$$\rho\frac{\mathrm{d}e}{\mathrm{d}t}=\mathbf{T}\cdot\mathbf{L}-\operatorname{div}\mathbf{q}+\rho\mathbf{r}, \tag{4.1}$$

where τ is the temperature, $\mathbf{q}$ $(=-K\partial\tau/\partial z$, K being the thermal conductivity) is the heat flux vector and $\mathbf{r}$ is the radial heating (taken here to be zero), e $(=C_p\tau$, where C_p is the specific heat) is the specific internal energy. The boundary conditions on the flow field are

$$\tau=\tau_1 \text{ at } z=-1, \quad \tau=\tau_2 \text{ at } z=1, \tag{4.2}$$

where τ_1 and τ_2 are temperatures of the disks.

Eq. (4.1) is now written as

$$\rho C_p\frac{\mathrm{d}\tau}{\mathrm{d}t}=K\frac{\mathrm{d}^2\tau}{\mathrm{d}z^2}+\frac{\mathrm{d}f}{\mathrm{d}z}s_{xz}+\frac{\mathrm{d}g}{\mathrm{d}z}s_{yz}, \tag{4.3}$$

where s_{xz} and s_{yz} are defined in Eqs. (2.15) and (2.16). Eq. (4.3) for steady case have the following form:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2}=-\frac{aE_cP_r}{c}\left[\left(\frac{\mathrm{d}\bar{f}}{\mathrm{d}\eta}\right)^2+\left(\frac{\mathrm{d}\bar{g}}{\mathrm{d}\eta}\right)^2\right], \tag{4.4}$$

where $\bar{f}=f/\Omega l$, $\bar{g}=g/\Omega l$, $P_r=\rho C_p\nu/K$ is the Prandtl number, $E_c=(\Omega l)^2/(\tau_1-\tau_2)C_p$ is the Eckert number and

$$\theta=\frac{\tau-\tau_2}{\tau_1-\tau_2},$$

where θ is the dimensionless temperature. We assume $\tau_1>\tau_2$, so that $E_c>0$ and which shows that heat is transferring from disk to the fluid.

From Eq. (3.9) we have

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\eta}\frac{\mathrm{d}\overline{\Gamma}}{\mathrm{d}\eta}=\left(\frac{\mathrm{d}\bar{f}}{\mathrm{d}\eta}\right)^2+\left(\frac{\mathrm{d}\bar{g}}{\mathrm{d}\eta}\right)^2, \tag{4.5}$$

where $\overline{\Gamma}$ is the complex conjugate of Γ. Using Eq. (4.5) in Eq. (4.4) we get

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2} = -\frac{aE_cP_r}{c}\frac{[(1+V_1)^2+V_2^2](\xi_1^2+\xi_2^2)}{\cosh(2\xi_1)-\cos(2\xi_2)} \times[\cosh(2\xi_1\eta)+\cos(2\xi_2\eta)]. \tag{4.6}$$

The boundary conditions on upper and lower disks when they are heated are

$$\theta(-1)=1, \quad \theta(1)=0. \tag{4.7}$$

The solution of Eq. (4.6) that satisfies the boundary conditions (4.7) is

$$\theta(\eta) = \frac{1}{2}-\frac{\eta}{2}+\frac{aE_cP_r}{c}\frac{\lfloor(1+V_1)^2+V_2^2\rfloor(\xi_1^2+\xi_2^2)}{\cosh(2\xi_1)-\cos(2\xi_2)} \times\left[\left\{\frac{\cosh(2\xi_1)-\cosh(2\xi_1\eta)}{4\xi_1^2}\right\} -\left\{\frac{\cos(2\xi_2)-\cos(2\xi_2\eta)}{4\xi_2^2}\right\}\right]. \tag{4.8}$$

5. The force and the torque

The component of the force exerted by the fluid on the bottom disk are

$$\begin{aligned} X &= \int_\lambda T_{xz}(-h)\,\mathrm{d}\lambda,\\ Y &= \int_\lambda T_{yz}(-h)\,\mathrm{d}\lambda,\\ Z &= \int_\lambda T_{zz}(-h)\,\mathrm{d}\lambda \end{aligned} \tag{5.1}$$

in which λ denotes the surface of the disk of radius r_0.

The force exerted by the fluid on the bottom disk is equal to that on the top disk. We have

$$\begin{aligned} X &= \pi r_0^2 s_{xz}(-h),\\ Y &= \pi r_0^2 s_{yz}(-h),\\ Z &= \frac{1}{2}\rho\pi\Omega^2 r_0^4\overline{Z}, \end{aligned} \tag{5.2}$$

where

$$\begin{aligned} s_{xz}(-h) &= \frac{\mu}{c}[af'-bg'],\\ s_{yz}(-h) &= \frac{\mu}{c}[bf'+ag'],\\ s_{zz}(-h) &= 0, \end{aligned} \tag{5.3}$$

$$\frac{1}{\Omega l}\frac{\mathrm{d}f(-h)}{\mathrm{d}z} = \frac{\begin{bmatrix}(1+V_1)\{\xi_1\cosh\xi_1\sinh\xi_1+\xi_2\cos\xi_2\sin\xi_2\}\\ -V_2\{\xi_2\cosh\xi_1\sinh\xi_1-\xi_1\cos\xi_2\sin\xi_2\}\end{bmatrix}}{\Delta}, \tag{5.4}$$

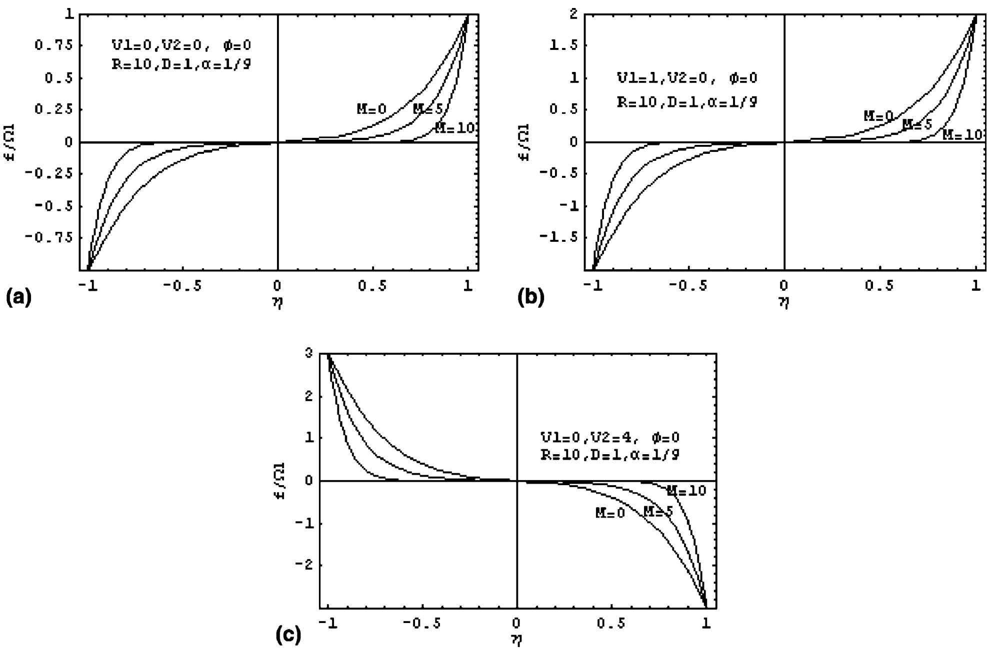

Fig. 1. The variation of $\bar{f}$ with η for $R = 10$, $D = 1$, $\alpha = 1/9$, $\phi = 0$, $M = 0, 5, 10$: $V_1 = 0$, $V_2 = 0$ in (a); $V_1 = 1$, $V_2 = 0$ in (b); $V_1 = 0$, $V_2 = 4$ in (c).

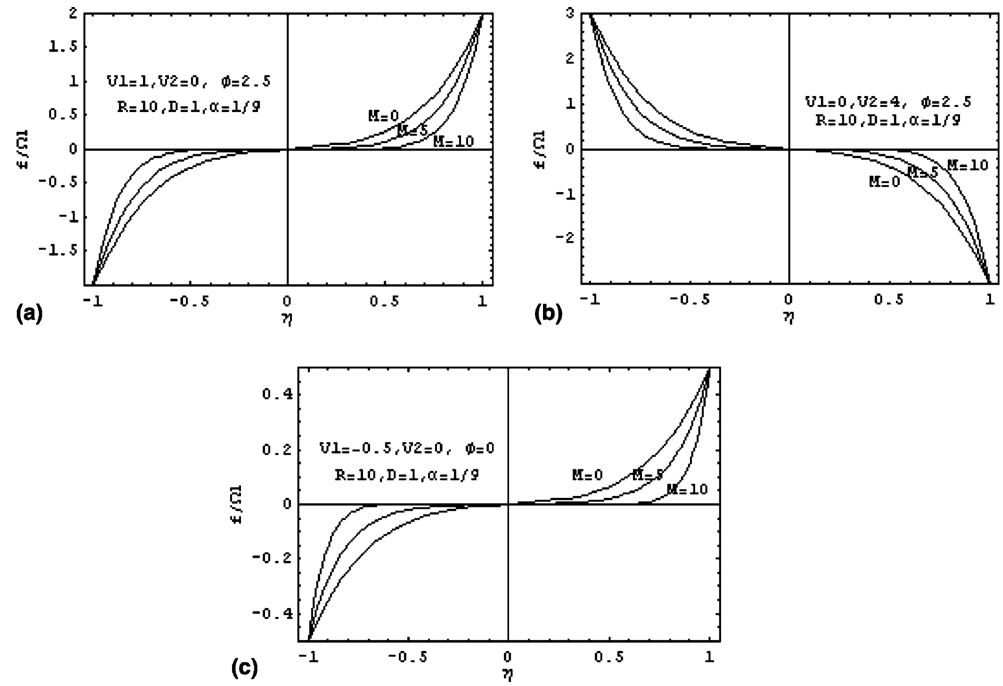

Fig. 2. The variation of $\bar{f}$ with η for $R = 10$, $D = 1$, $\alpha = 1/9$, $M = 0, 5, 10$: $V_1 = 0$, $V_2 = 0$, $\phi = 2.5$ in (a); $V_1 = 0$, $V_2 = 4$, $\phi = 2.5$ in (b); $V_1 = -0.5$, $V_2 = \phi = 0$ in (c).

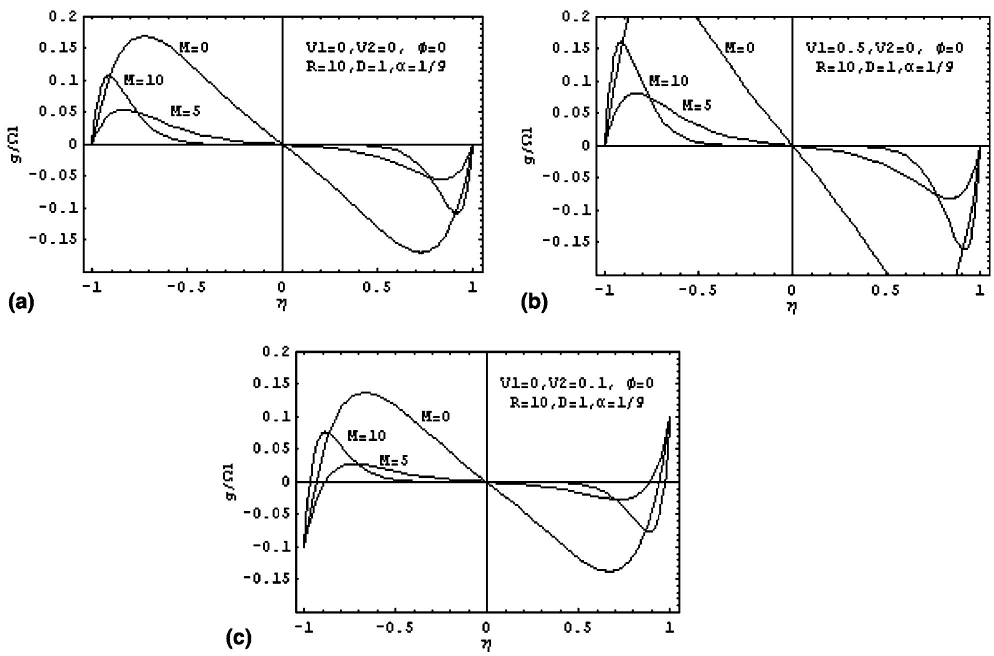

Fig. 3. The variation of $\bar{g}$ with η for $R = 10$, $D = 1$, $\alpha = 1/9$, $\phi = 0$, $M = 0, 5, 10$: $V_1 = 0$, $V_2 = 0$ in (a); $V_1 = 0.5$, $V_2 = 0$ in (b); $V_1 = 0$, $V_2 = 0.1$ in (c).

$$\frac{1}{\Omega l}\frac{\mathrm{d}g(-h)}{\mathrm{d}z} = \frac{\begin{bmatrix}(1+V_1)\{\xi_2\cosh\xi_1\sinh\xi_1-\xi_1\cos\xi_2\sin\xi_2\}\\ +V_2\{\xi_1\cosh\xi_1\sinh\xi_1+\xi_2\cos\xi_2\sin\xi_2\}\end{bmatrix}}{\Delta}, \tag{5.5}$$

$$\overline{Z} = -\left[\frac{\rho_0}{\frac{1}{2}\rho\Omega^2 r_0^2}+\frac{1}{2}\right]. \tag{5.6}$$

On the bottom disk the torque exerted by the fluid is

$$\chi = \int_\lambda [xT_{yz}(-h) - yT_{xz}(-h)]\,\mathrm{d}\lambda. \tag{5.7}$$

Since $T_{yz}(-h) = s_{yz}(-h)$ and $T_{xz}(-h) = s_{xz}(-h)$ it follows that

$$\chi = 0. \tag{5.8}$$

6. Numerical results and discussions

The unknown functions $f/\Omega l$ and $g/\Omega l$ given by Eqs. (3.11) and (3.12) in velocity components are plotted against η in Figs. 1–4. Also the dimensionless temperature is displayed against η in Figs. 5 and 6. Of particular interest here are the effects of M, R, D, α, ϕ, E_c, P_r, V_1 and V_2. In all profiles drawn in Figs. 1–4 we fixed ($R = 10$); ($D = 1$); $\alpha = 1/9$; and varied $M = 0, 5, 10$;

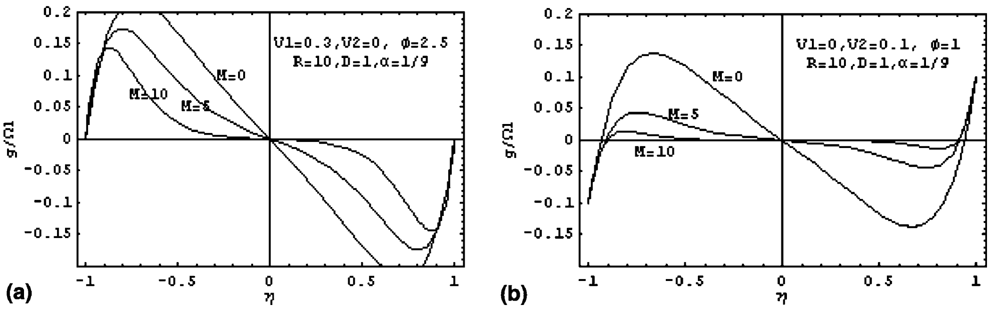

Fig. 4. The variation of $\bar{g}$ with η for $R = 10$, $D = 1$, $\alpha = 1/9$, $M = 0, 5, 10$: $V_1 = 0.3$, $V_2 = 0$, $\phi = 2.5$ in (a); $V_1 = 0$, $V_2 = 0.1$, $\phi = 1$ in (b).

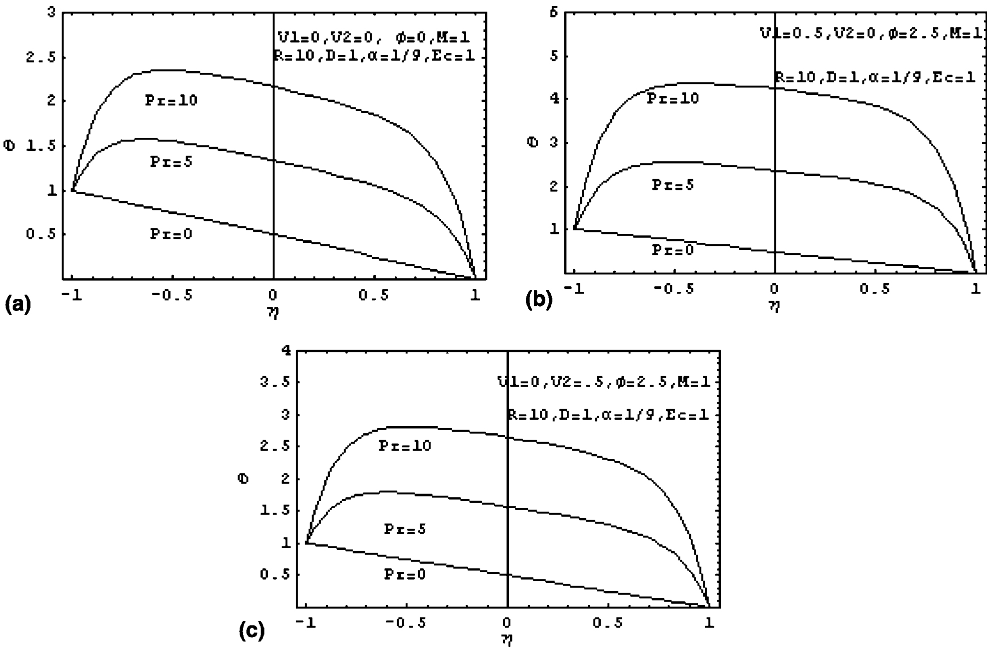

Fig. 5. The variation of θ with η for $R = 10$, $D = 1$, $\alpha = 1/9$, $M = 1$, $E_c = 1$, $P_r = 0, 5, 10$: $V_1 = 0$, $V_2 = 0$, $\phi = 0$ in (a); $V_1 = 0.5$, $V_2 = 0$, $\phi = 2.5$ in (b); $V_1 = 0$, $V_2 = 0.5$, $\phi = 2.5$ in (c).

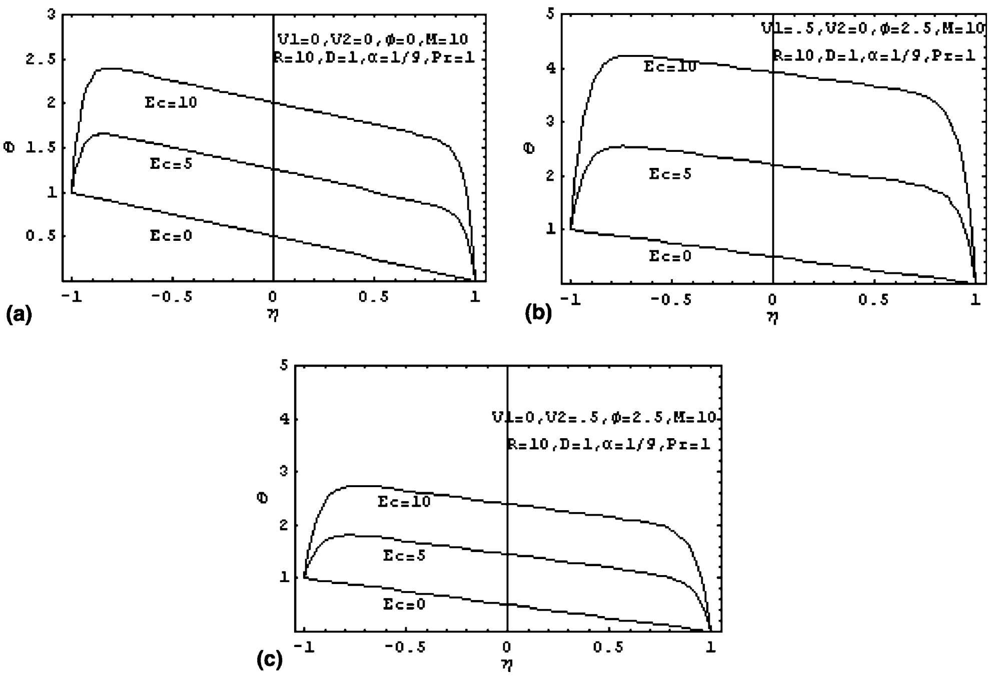

Fig. 6. The variation of θ with η for $R = 10$, $D = 1$, $\alpha = 1/9$, $M = 10$, $P_r = 1$, $E_c = 0, 5, 10$: $V_1 = 0$, $V_2 = 0$, $\phi = 0$ in (a); $V_1 = 0.5$, $V_2 = 0$, $\phi = 2.5$ in (b); $V_1 = 0$, $V_2 = 0.5$, $\phi = 2.5$ in (c).

$\phi = 0, 1, 2.5$; $V_1 = 0, 0.3, 0.5$; and $V_2 = 0, 0.1, 4$. It is found in Fig. 1a and b that when V_1 varies from 0 to 1 then $f/\Omega l$ varies from $(-1 \to 1)$ to $(-1 \to 2)$. Further, Fig. 3a and b indicate that $g/\Omega l$ is large when V_1 is increased from 0 to 0.5. It is further observed from Fig. 2c that $f/\Omega l$ and $g/\Omega l$ decrease when V_1 is negative. Also, similar observations hold in Figs. 1c and 3c when $V_2 = 4$ and $V_2 = 0.1$, respectively. In Figs. 1–4, the effect of M on the velocity profiles is also taken into account. It is concluded that the applied magnetic field tends to decelerate the layer thickness. Figs. 1c and 2b and 3b and 4a indicate that for large $\phi = 0 \to 2.5$ the layer thicknesses in $f/\Omega l$ and $g/\Omega l$ increase.

The dimensionless temperature distributions θ versus η are plotted in Figs. 5 and 6 for different values of E_c, P_r, ϕ, V_1, V_2, E_cP_r, and fixed $R = 10$, $D = 1$, $\alpha = 1/9$, and $M = 1$. In Fig. 5, $E_c = 1$, $P_r = 0, 5, 10$, $V_1 = 0, 0.5$, $V_2 = 0, 0.5$, and $\phi = 0, 2.5$. For $P_r \geqslant 0$ and $E_c = 1$ we get $P_rE_c \geqslant 0$ and thus θ increases for large values of P_r. Also it is observed from Fig. 5a and b or Fig. 5a and c that for large values of disk velocities $V_1 = 0.5$, $V_2 = 0$ and $V_1 = 0$, $V_2 = 0.5$, respectively, θ increases and vice versa. Fig. 6 is sketched for $P_r = 1$, $M = 10$, and $E_c = 0, 5, 10$. It is obvious from Fig. 6 that θ increases with Eckert number, V_1 and V_2. Thus it is concluded from Fig. 6 that when the product $P_rE_c \geqslant 0, 5, \ldots,$ the velocity increases and thus the boundary layer thickness decreases.

7. Conclusion

In this paper the effects of Hall current and heat transfer are analyzed on the flow of an Oldroyd-B fluid between eccentric rotating disks. The governing equations resulting from the momentum and energy laws are solved analytically to examine the sensitivity of the flow to the parameters that are used in the modeling of the problem. From the presented analysis, the work findings can be summed up as:

1. It is found that with an increase in V_1 and V_2 the velocity increases and with the decrease in V_1 and V_2 the velocity decreases. The same is true for the heat transfer case.
2. With an increase in Hartmann number M the velocity increases and hence the boundary layer thickness decreases.
3. It is worth mentioning that the layer thicknesses increase with the Hall parameter ϕ for fixed Hartmann number M although decrease with increase in M for a fixed Hall parameter ϕ.
4. When the magnetic Reynolds number is very small, the flow pattern with Hall effects is remarkably similar to that for non-conducting flow. Of course, the assumption of very small magnetic Reynolds number will be valid for flow of liquid metals or slightly ionized gas. For a slightly ionized gas, the electron pres-

sure gradient term in Eq. (2.7) is negligible. However, for a fully-ionized gas, the last term in Eq. (2.7) is significant with $p_e = p/2$, while the ion-slip term can be ignored.
5. $\phi \to \infty$ will give the result of hydrodynamic case and $\phi = 0$ gives us the results for the magnetohydrodynamic case.
6. The Hall current also contribute to steady flow when an Oldroyd-B fluid is not identical to Newtonian fluid.
7. The increase in $P_r(0, 5, 10)$ and fixed $E_c = 1$ increases the velocity. When P_r is small the effect of viscous dissipation E_c is very small.
8. It is noted that the smaller the value of the Prandtl number the thicker is the steady state thermal boundary layer and hence the longer it takes for the temperature to reduce to its ambient value.
9. The thermal boundary layer thickness increases with the viscoelastic effects as the momentum boundary layer thickness does, while it decreases with increase in P_r.

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